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TWISTING MOMENT AND FLOOR SLAB DESIGN[1]

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This Technical Note comments on the role of twisting moment in design of floor slabs.

- Floors are designed primarily for the two conditions: (i) service (SLS) and (ii) safety (ULS).

SERVICE CONDITION

- The central checks for service condition are (i) deflections, and (ii) formation and extent of probable cracking, if any.
- The in-service response of a floor to applied loads, including moderate local cracking is adequately represented by the floor's linear elastic response.
  - The elastic response of a floor for the range of deflections common in concrete structures is expressed by "theory of elastic plates."
  - The plate's (floor's) internal mechanism of resistance to the applied load under service condition is by way of:
    - Direct moments (Mxx and Myy about two orthogonal axes)
    - Twisting moments (Mxy and Myx about the same axes as the moments)
    - Shear normal to the plate's surface (Qx and Qy)
      - Direct moments (Mxx and Myy) result in "axial stresses" over the cross-section of a slab. Axial stresses are those that we generally refer to as "tension" or "compression." – like pull and push. We check the magnitude of axial tension to determine the likelihood, and extent of probable cracking in slabs.
      - The twisting moment (Mxy) results in "shear stresses" in the "plane of the slab." The shear stresses from the twisting moments are in direction of the plane of the slab. These shear "stresses" are not additive the bending "stresses." They are normal to one another. More importantly, they do not contribute to cracking that can be associated to "Mx, or My."
      - Normal shear forces (Qx and Qy) result in shear stresses perpendicular to the plane of a slab.
- For in-service design of a slab, linear elastic theory is used to (i) determine the deflections, and (ii) moments Mxx and Myy. Moments Mxx, and Myy cause tension and compression. These moments are used as "entry values" to evaluate the potential and extent of cracking in service. Stress at a "point" for the "non-homogeneous" nature of concrete, having aggregates larger than a "point" and embedded with rebar is not a reliable pointer for crack evaluation. To predict a slab's behavior for design-significant crack formation, crack length and crack width, a "finite length" of slab, along with the sum of moments acting over that finite length are considered. The selection of a finite length is understood to smear the inherent non-homogeneity of a concrete

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slab and local peaks in moment values, leading to a better correlation with the response of the prototype. In most applications, a length not less than five times the slab thickness is appropriate. However, for a general, uniform, and simplified procedure to be used by designers, major building codes recommend the width of the control section to be that adopted for "safety" check of a slab, namely design strips and design sections – with recommended stress values adjusted to account for the wider strips.

- The preceding discussion leads to the following conclusions regarding the serviceability check of a concrete floor.
  - In deflection calculation of floors, the contribution of twisting moment is fully accounted for. Twisting moments are duly included in the formulation of elastic theory of plates. The contribution of twisting moments in deflection and stress calculation is implicit in the solutions. No subsequent inclusion of twisting moments is warranted.
  - For evaluation of probable formation and extent of cracks, the direct moments Mxx, and Mxx along with a "finite" width of slab apply. Again, the contribution of twisting moments is included in the underlying formulation that leads to the values of the direct moments. No subsequent accounting for twisting moments is necessary. Further, the computed stress values at a "point" do not reflect the response of a slab in use.

SAFETY CHECK

- The first step in safety check of a floor slab is to envisage how the slab at hand is likely to fail. We call this "failure mechanism."
- The second step is to make sure that at the envisaged failure mode, the slab develops adequate strength - not less than that demanded by the "design" loads.

- We start by reviewing the first step, namely the "failure mechanism" of a floor slab.
  - Consider a steel wire hanging and holding a weight W (Fig. 1a). The extension of the wire under the weight reflects its in-service response. The extension is governed by the uniaxial tension generated in the wire. In terms of our current discussion, the mechanism of load transfer from the point of application of the weight to the support of the wire at top is by way of a uniform axial force (stress) along the wire.
- Moving to the next step, namely the safety check of the wire structure, we have to envisage a failure mechanism. In this case, increase in extension of the wire will lead to its rupture. At failure the mechanism of load transfer is again an axial tension along the length of the wire. It is the "same mechanism" as the one governing the service condition. Simply, the magnitude of the force at failure is larger. For this structure we make the following observations.
  - For a hanging wire, the mechanism of load transfer for service condition "coincides" with that of safety condition. There is a "single" mechanism of load transfer. At ULS, the member does not change its way of resisting the load. Simply, the value of the member's resisting forces (stress in this case) become larger, until the capacity of the member is exhausted.
- Next, consider a single slab panel supported on four walls and subjected to a uniformly distributed load (Fig. 1b).
  - At low values of load, under service condition, the deflection of the slab is governed by theory of elastic plates. As discussed above, at each location, the slab develops direct moments, twisting moments and shears to resist the load. A bending moment distribution in the form outlined by the contour lines of Fig. 1c is appropriate.
  - Increasing the load, the slab failure can take place in a "multitude" of configurations, depending on how the slab is reinforced.
    - If the slab is reinforced only in one-direction, the failure mechanism will be as shown in Fig. 1-d. The demand actions along the failure hinge line consist of direct moments (M), for which adequate reinforcement shall be provided. For this failure mechanism, the twisting moments Mxy play no role either in the

determination of design values, or the computation of reinforcement. The strength mechanism is different from that of the service condition, and is governed by its own rules.

- Consider the hinge formation shown in Fig. 1-e for a biaxial failure mechanism based on a two-way layout of reinforcement. The location and amount of the reinforcement, determine the position and extent of the hinges. The method of computation of demand values is well documented in the literature[3]. While it is recognized that twisting moments were contributory in the initial deflection of the floor, in this case the twisting moments do not play a role either in the determination of design values, or the provision of reinforcement. The failure mechanism at ULS shown is different from that of SLS for which twisting moments were contributory.
- While, based on the foregoing, contemporary practice of floor slab design does not call for inclusion of Mxy for strength condition of a floor slab, it is possible to do so, should a designer so desires. The inclusion of Mxy in ULS should be applied to a failure mechanism for which Mxy is a contributory component. This can be achieved, but it is not practiced. There are two options for construction of a floor slab, for which the inclusion of twisting moment is applicable. These are:
  - One option is the application of uniformly distributed fibers dispersed throughout the volume of a slab, with neither rebar nor post-tensioning. The arrangement of fibers shall be such as to create a strength distribution matching that of the slab's elastic response. At incipient failure, the combination of direct and twisting moments will reach the local capacity over the entire slab. Assuming ductility, a change in the distribution of resistance by the fibers in the slab, will result in a re-distribution of demand moments and a different failure mechanism. Any "Redistribution" of design values prior to failure, invalidates the elastic plate theory - the cornerstone for determination of twisting moments. "Redistribution" brings with it new rules based on static equilibrium and ductility. Obviously, the example of slab with distributed fibers a described is not a practical scenario. If there is ductility, and the demand values re-distribute to cause a failure mechanism other than that dictated by the elastic distribution of forces, again, in line with conditions expressed for Figs. 1-d and 1-e, other modes of computation apply. The inclusion of twisting moments for safety check is based on providing resistance at each "point" of a slab to be "exactly equal" to demand generated at that point, and no ductility for subsequent re-distribution.
  - The second option is to provide closely spaced top and bottom reinforcement in both directions over the entire area of a slab. The spacing of the reinforcement shall not exceed slab thickness, in order to be available in providing resistance at the point of initial demand. At each location, the reinforcement shall be designed to resist the combination of direct and twisting moments at that point. In this case the Wood-Armer combination of direct and twisting moments may be used. The slab is deemed to fail over its entire area contemporaneously by reaching the capacity of the distributed reinforcement over the entire floor area of slab at a given value of applied load. Provision of ductility and re-distribution of demand moment will lead to a different failure mechanism which invalidates the distribution of demand values used as basis of design.

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[3] Nawy, E. G., (1997), "Prestressed Concrete, A Fundamental Approach," Prentice Hall, International, New Jersey,
3rd ed., 938 pp. 1997.

- The contemporary method of design for floor slabs determines the initial demand for resistance using the linear elastic theory of plates[4]. Using load paths (design strips and design sections) selected by design engineers, the distribution of actions determined from the elastic solution is channeled to actions along the selected load paths. Each load path is provided with adequate reinforcement to resist the associated design values. For each load path, the demand values are calculated using all six components of actions, including twisting moments that are germane to elastic solution[5]. Ductility provided in design guarantees the re-distribution of actions to where resistance is provided (design strips and design sections).

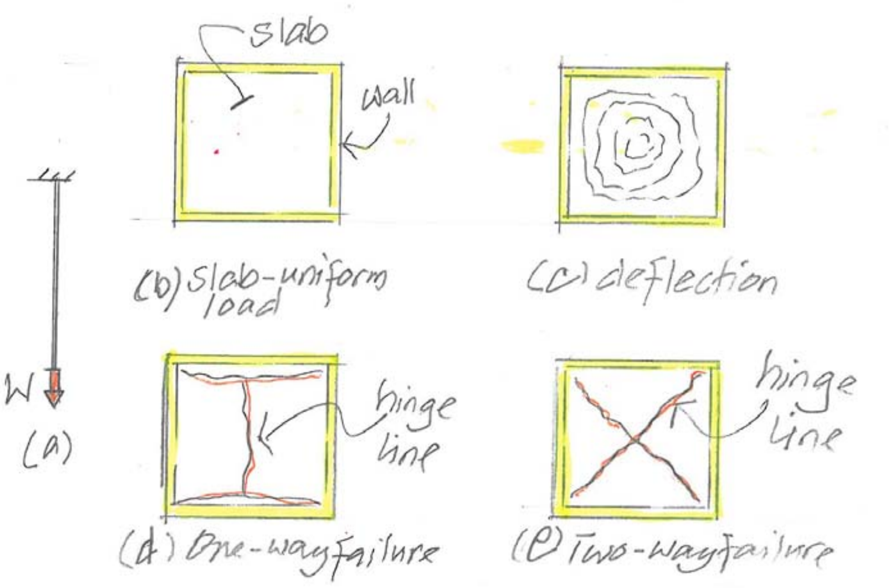

FIGURE 1 – Wire String and Floor Slabs Under Load

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Notwithstanding the foregoing, since engineering practice is a combination of science, art, opinion, following one's peers, and achieving a sense of comfort, for safety check of floor systems, a number of design tools[6] provide the option of increasing the absolute value of direct moments by the value of twisting moments, when reporting the design moments. The objective of the tool is to provide the engineers with the sense of comfort and support the habit. As long as the added value in moment results in increase of reinforcement – not prestressing – the practice generally does not impact the structural performance of a slab adversely. Simply, it results in reinforcement beyond that necessary for safety of the structure.

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[4] ADAPT-Floor Pro www.adaptsoft.com
[5] Three moments and three forces combined with the application of extended nodal integration
[6] ADAPT-Floor Pro, www.adaptsoft.com